Approximate reasoning with type-2 fuzzy set

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SYNOPSIS

In the year 1965, Zadeh formally introduced the concept of fuzzy set. Over the years, a comprehensive theory of fuzzy sets has been developed with the contribution of many researchers. The successful application of the theory proved that it is an extraordinary mathematical tool for representing imprecise human knowledge. In doing so, Zadeh himself observed that in many decision- making processes knowledge could be better represented by means of a generalized version of fuzzy sets [9]. Accordingly, in the year 1973. Zadeh proposed an extension of a fuzzy set and called it an interval-valued fuzzy set where the membership degree of an element to belong to a fuzzy set is represented as a closed sub-interval of the interval [0,1]. It is expected that with such a representation, vagueness (lack of sharp class boundaries) and uncertainty (lack of information) could be addressed simultaneously with in a single framework.

On perusal of existing literature it has been found that type-2 fuzzy sets are such that membership value of an element to belong to such a set is a fuzzy subset over [0,1] instead of a crisp number in [0,1], representing the uncertainty about the degree of membership of an element in a fuzzy set. Thus, by nature, type-2 fuzzy set could be useful in circumstances where it is difficult to determine the exat membership function for an interval to belong to a fuzzy set. Membership functions of type-2 fuzzy sets are itself fuzzy. This research proposes to study type-2 fuzzy set to express uncertainty and vagueness inherent in fuzzy systems in a natural way [2,3,5].

Once a formal framework for studying the operations on fuzzy sets of type-2 is understood and/or developed it is proposed to study approximate reasoning in presence of type-2 fuzzy sets. For that a comprehensive study of binary relations induced by two or more type-2 fuzzy sets is to be undertaken. One basis for all these studies could be Zadeh's extension principal. It is hope that composition of relations, extension, projection, conjunction, complementation, modification etc. [1,6,8] could then be for the development of a comprehensive theory of approximate reasoning using fuzzy sets of type-2.

It is already been established that type-2 fuzzy set models and minimizes the effects of uncertainty in rule based fuzzy logic systems. Unfortunately, type-2 fuzzy sets are more difficult to use and comprehend. Consequently, their use in designing fuzzy systems has been rare. This work seeks prove that once a comprehensive theory of type-2 fuzzy sets is developed an approximate reasoning methodology with type-2 fuzzy sets could be modified/framed. The existing theory of fuzzy logic based control could be revisited.

To achieve the goal of the proposed research work, one fundamental object is to develop an algebra of truth-values of sentences represented using type-2 fuzzy sets. It has been observed that such an algebra could successfully be applied in the manipulation of the operations defined on the sets. These induced operations on the set of fuzzy subsets of type-2 defined over a universal set would satisfy the same relation/equations as the

corresponding operations on the original algebra of the truth-values. Consequently, additional operations defined on [0,1] would provide analogical definition of the same operations on fuzzy subsets of [0,1]. These motivates new definitions of t-norm and t- conorms which are found to play an important roll in any logical development of approximate reasoning methodology [7] . in fact, the fundamental operations on fuzzy sets of type-2 could be redefine in terms of t-norm / t- conorm operations. This research proposed to study t-norm like operations on fuzzy subsets of [0,1]. Consequently, the implication and conjunction operations used in representing the uncertain rules in a rule-based fuzzy system is to be redefined. For generating logical consequences, rule of inferences are to be modified accordingly. In the process, the concept of similarity is to be considered. Similarity measures for fuzzy sets of type-2 is to be proposed. Role of similarity relations is to be extensively studied for the definitions of fuzzification and defuzzification procedures used in fuzzy logic systems.

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